Diffraction of light

Diffraction at a Straight Edge

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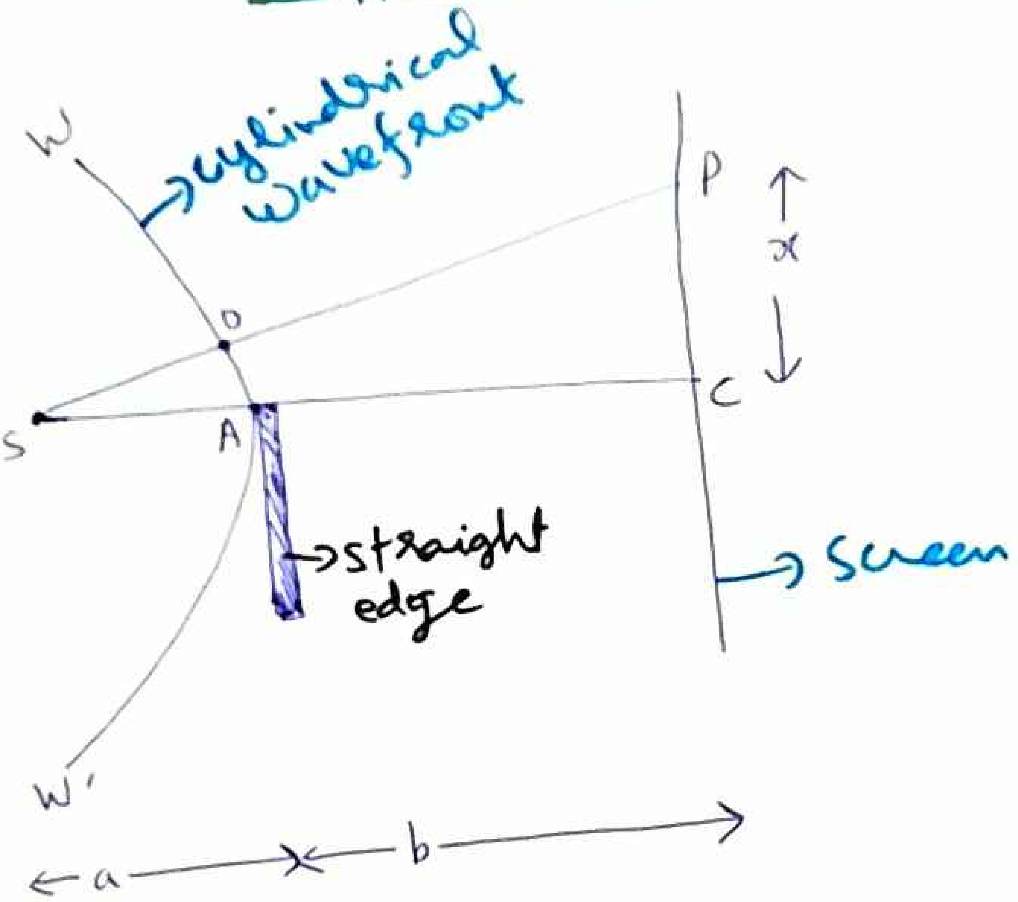


Diffraction Pattern consists of a series of maxima and minima parallel to the edge of the straight edge in the region immediately above C. These maxima and minima are not equidistant (unlike inferefarence fringes) but gradually become closer as we move above C. Also, intensity of the maxima decreases while that of minima increases as we move above C (Actually, just above C, intensity is about 2.25 times greater than that of due to unobstracted wave front.). After a short distance from C, uniform illumination is observed.

Some of the light also penetrates into the geometrical shadow and the illumination

falls off continuously and rapidly to zero, (2)
without showing any maxima or minima.

To explain the characteristics of the diffraction pattern, we shall divide the cylindrical wavefront emerging from the slit into half period strips.

Let WW' be the section of the wavefront when it just touches the edge A. Let P be any point on the screen on which intensity has to be determined.

Joint P and S.

Suppose this line cuts the wavefront WW' at O. Thus the wavefront is divided into two halves - OW and OW'.

Now, with P as centre and radii $PO + \frac{\lambda}{2}$, $PO + \frac{2\lambda}{2}$, $PO + \frac{3\lambda}{2}$, ---- etc draw the circles intersecting the section WW' at different points. Through these points, lines are drawn // to the length of the slit. The wavefront is thus divided into half period strips.

Resultant Amplitude at P - due to each half of the wavefront is given by $R_1/2$, where R_1 is the amplitude at P due to the first half period strip in either halfs of the wavefront.

Uniform illumination at points far above C :- ③

When P is far above C, then OA contains all the effective half period strips of the wavefront OW'.

$\therefore$ Contribution of OA portion of the wavefront is also $R_1/2$.

$\therefore$ Resultant amplitude at P $= \frac{R_1}{2} + \frac{R_1}{2}$

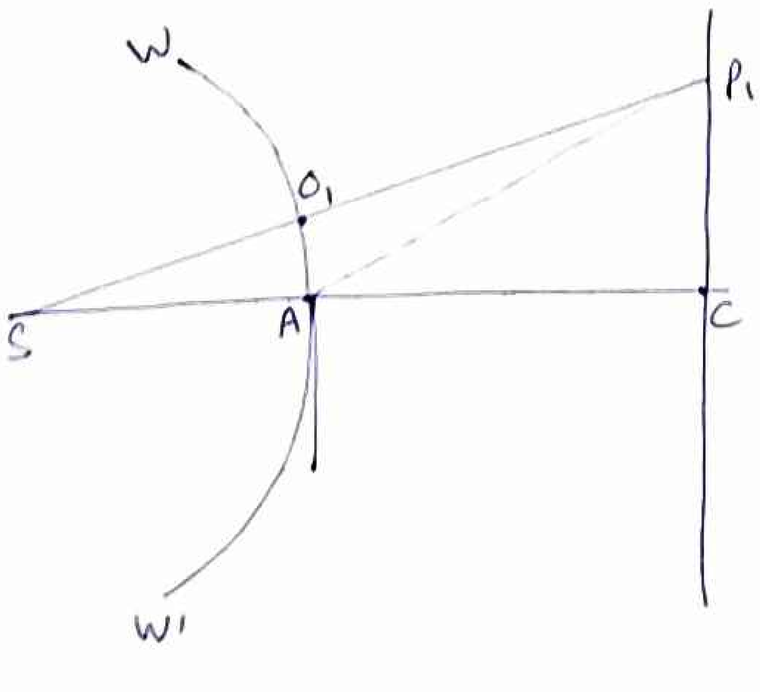
due to OW'   due to OW

$$\boxed{= R_1}$$

= amplitude at P due to the unobstructed wavefront.

Therefore we have uniform illumination at these points.

Maxima and Minima Regions -

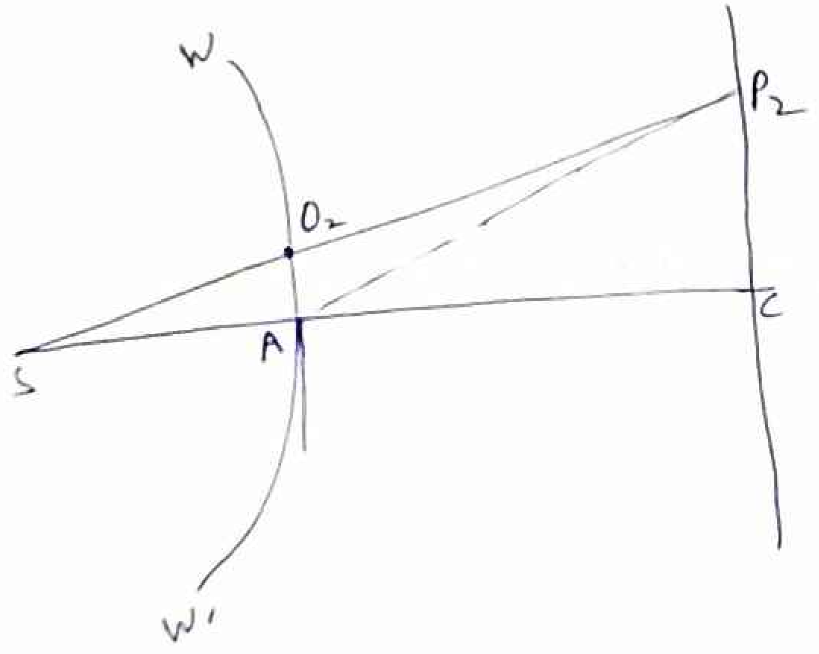


Suppose point P_1 is such that the relation $P_1A - P_1O_1 = \frac{\lambda}{2}$ is satisfied. Then O_1A will contain only first half period strip of the half wavefront O_1W'.

$\therefore$ amplitude at P_1 = amplitude due to the portion O_1W ~~and~~ + amplitude due to the first half period strip of wavefront O_1W'.

i.e. $D_1 = \frac{R_1}{2} + R_1 = \frac{3}{2} R_1$

$\therefore$ intensity at $P_1 = \frac{9}{4} R_1^2 = 2.25 R_1^2$, which is 2.25 times greater than the intensity at P_1 due to the unobstructed wavefront.



as we move above C, we come across a point P_2 such that $P_2A - P_2O_2 = \frac{2\lambda}{2}$

Then O_2A contain I and II half period strips of the wavefront portion O_2W'.

$\therefore$ amplitude at P_2 = amp. due to O_2W + amp. due to O_2A part of O_2W'

or $D_2 = R_1/2 + (R_1 - R_2) = \frac{3R_1}{2} - R_2$

$\Rightarrow$ Intensity at P_1 > Intensity at P_2

$\Rightarrow$ Maxima at P_1 is followed by a minima.

(But this minima is not absolute because D_2 has some +ive value).

Further above, we come across a point P_3 such that $P_3A - P_3O_3 = \frac{3\lambda}{2}$

Then O_3A will contain I, II and III half period strip of O_3W'.

$\therefore$ Amplitude at P_3, $D_3 = \frac{R_1}{2} + R_1 - R_2 + R_3$

Similarly at P_4, such that

$$P_4A - P_4O_4 = \frac{4\lambda}{2}, \text{ we have}$$

$$D_4 = \frac{1}{2}R_1 + R_1 - R_2 + R_3 - R_4$$

Now, we can write,

$$R_2 \approx \frac{1}{2}(R_1 + R_3)$$
$$R_4 \approx \frac{1}{2}(R_3 + R_5) \text{ etc.}$$

($\because$ magnitudes of $R_1, R_2, R_3 \cdots$ are decreasing successively).

Thus we have, $D_1 = R_1 + \frac{R_1}{2}$

$$D_2 = \frac{3R_1}{2} - \frac{1}{2}(R_1 + R_3) = R_1 - \frac{1}{2}R_3$$

$$D_3 = \frac{R_1}{2} + R_1 - \frac{1}{2}(R_1 + R_3) + R_3 = R_1 + \frac{R_3}{2}$$

$$D_4 = \frac{1}{2}R_1 + R_1 - \frac{1}{2}(R_1 + R_3) + R_3 - \frac{1}{2}(R_3 + R_5)$$
$$= R_1 - \frac{R_5}{2}$$

$D_5 = R_1 + \frac{R_5}{2}$ etc

Now since, $R_1 > R_2 > R_3$ etc

$\therefore D_3 > D_2$ and $D_3 > D_4$

$\Rightarrow D_3$ is brighter than D_2 and D_4

$\Rightarrow$ A maxima is obtained at P_3 followed by a minima on either side.

Similarly we can show that P_1, P_3, P_5, P_7 etc corresponds to the region of maximum intensity while $P_2, P_4, P_6 \ldots$ corresponds to minimum intensity.

Also, since $D_1 > D_3 > D_5 > D_7$ etc

and $D_2 < D_4 < D_6$ etc

$\Rightarrow$ intensity of the maxima decreases gradually while that of minima increases.

From this analysis, it is obvious that intensity at any point P will be maximum or minimum according as OA exposes odd or even no of half period strips of OW. Mathematically, conditions of maxima and minima can be expressed as

$$PA - PO = (2n+1)\frac{\lambda}{2} \quad \text{(Maxima)}$$

$$PA - PO = n\lambda \quad \text{(minima)}$$

Positions of Maxima and minima at the screen can be easily calculated.

From the figure on Page 1,

$$PA^2 = x^2 + b^2$$

$$\therefore \quad PA \approx b\left(1+\frac{x^2}{b^2}\right)^{1/2} \approx b + \frac{1}{2}\frac{x^2}{b}$$

Similarly, $PS \approx \sqrt{(a+b)^2+x^2} \approx (a+b) + \frac{x^2}{2(a+b)}$

$$\therefore PO = PS - SO = b + \frac{x^2}{2(a+b)}$$

$$\therefore PA - PO = \frac{ax^2}{2b(a+b)} \quad \text{--- (1)}$$

$\therefore$ Points of maximum intensity are given by

$$\boxed{\frac{a x_{max}^2}{2b(a+b)} = (2n+1)\frac{\lambda}{2}}$$

which gives $x_{max} = K\sqrt{2n+1}$ --- (2)

similarly, $x_{min} = K'\sqrt{n}$ --- (3)

$\therefore$ For first maxima $x_1 = K$ (for $n=0$ in eq (2))

for II maxima $x_2 = K\sqrt{3}$

for III " $x_3 = K\sqrt{5}$

$\therefore$ separation between consequtive maxima will be $x_2 - x_1 = 0.732\,K$

$$x_3 - x_2 = 0.504\,K$$

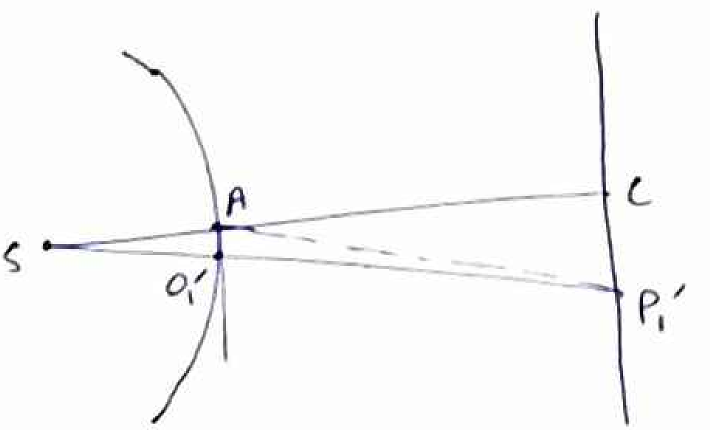
$$x_4 - x_3 = 0.430\,K$$

$\Rightarrow$ Maxima get closer as we move above the edge of the geometrical shadow. (8)

Intensity at the edge of the geometrical shadow – If point P lies at C then only upper half of the wave is exposed and the lower half is completely obstructed. Therefore resultant amplitude at $C = R_1/2$.

$\therefore$ intensity at $C \propto R_1^2/4$, which is simply $\frac{1}{4}$th of the intensity produced by the unobstructed wavefront. Also intensity at C is less than that at first maxima point P_1.

Penetration of light with in the geometrical shadow –



Consider the point P_1' on the screen below C, such that

$$P_1'A - P_1'O_1' = \frac{\lambda}{2}$$

Then for this point, obstacle not only cuts off completely the lower half of the wavefront but also the first half period strip of the upper wavefront. Therefore resultant amp. at P_1' is $D_1' = R_2 - R_3 + R_4 \cdots = \frac{R_2}{2}$

and intensity at $P_1' \propto R_2^2/4$, which is less than that at C. (9)

For point P_2' below C, such that

$$P_2'A - P_2'O_2' = \lambda, \text{ I and II}$$

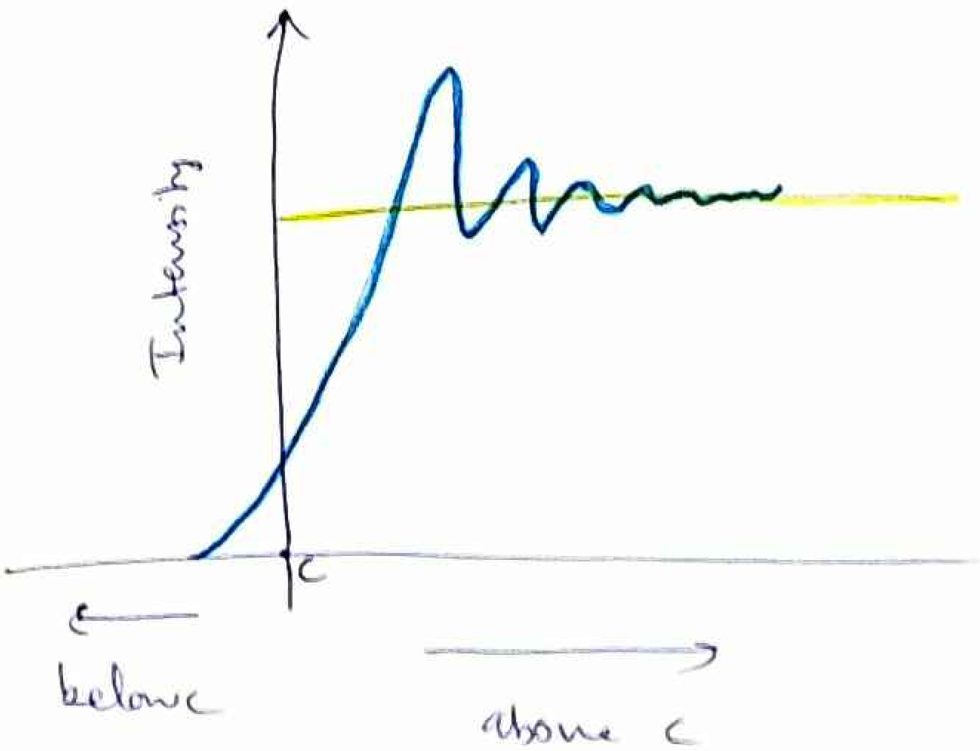
half period zones of OW are also blocked

$$\therefore D_2' = R_3 - R_4 + R_5 - - -$$
$$= R_3/2$$

and intensity at $P_2' \propto R_3^2/4$

$\Rightarrow$ intensity at P_2' < intensity at P_1'

similarly we can show that intensity falls off continuously and rapidly without showing any maxima or minima.



(Intensity contour)